equivalent circuit. Wall losses are taken into account by perturbational methods. Total wall losses are separated into dominant and excess losses. Dominant losses are included in a lossy transmission-line model. Excess losses may be neglected or incorporated by further modification in the equivalent circuit.

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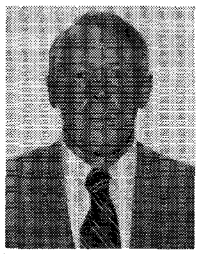

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Complex Propagation Constants of Bent Hollow Waveguides with Arbitrary Cross Section

MITSUNOBU MIYAGI

Abstract—**An integral representation of the complex propagation constant β has been derived from Maxwell's equations for cylindrical, hollow, bent, oversized waveguides with uniform curvature and with arbitrary cross sections. The method makes the calculations much simpler than the conventional method, i.e., the characteristic-equation method, although it has not yet been tried for three-dimensional bent waveguides.**

Manuscript received April 18, 1984; revised July 17, 1984.
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I. Introduction

FOLLOW WAVEGUIDES are important transmission media for CO_2 laser light because they are expected to be able to carry high power [1]. One of the serious problems of hollow waveguides is the increased loss due to bends. Therefore, waveguide structures with small bending losses should be designed for the realization of a high-powered delivery system [1]–[4].

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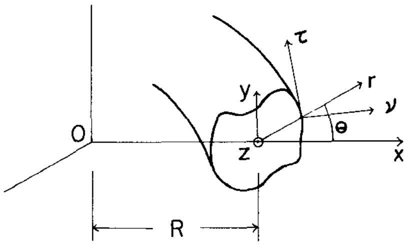

Fig. 1. The coordinate system for bent waveguides with a uniform bending radius R.

In order to evaluate bending losses in circular metallic or dielectric hollow waveguides, a theory presented by Marcatili and Schmeltzer [5] has been used for the past two decades at infrared as well as submillimeter wavelengths [6], [7]. To evaluate losses, they used a series expansion method to evaluate field deformations and obtained the power-attenuation constant as the ratio of P_l/P_z, where P_l is the power lost per unit length and P_z is the power carried by a given mode. However, as pointed out in a previous paper [8], they didn't consider field deformations depending on R^{-2} (R: bending radius), which yields wrong bending loss formulas.

To study wave propagation in a circular metallic waveguide with infinite conductivity, a series expansion method for the field deformations and the propagation constant was also employed in the book by Lewin [9]. However, he only mentioned that the coefficient of the propagation constant depending on R^{-1} is zero, and no expression was presented for the propagation constant depending on R^{-2}.

On the other hand, in order to study wave propagation in a rather general class of hollow waveguides, applicable to oversized waveguides with finite conductivity, the concept of wall impedance was introduced by Karbowiak [10] and was used by Dragone [11], [12] and Lindell [13] for studying oversized, arbitrarily shaped waveguides.

In this paper, the complex propagation constants of bent oversized waveguides are studied using the wall impedance method. The previous theory [8] has been extended to waveguides with arbitrary cross sections and a uniform bending radius R. When the complex propagation constant β is approximated by $\beta_0+\delta\beta_1/R+\delta\beta_2/R^2$, $\delta\beta_1$ can be evaluated from only the zeroth-order field distributions, and $\delta\beta_2$ can be evaluated from fields depending on R^0 and R^{-1} but not on R^{-2}, which makes calculations extremely simple.

II. Analysis

Consider a hollow waveguide with arbitrary cross section bent with a bending radius R as shown in Fig. 1. For convenience, we employ a toroidal coordinate system (r, θ, z) and borrow most of results given in the previous paper [8]. The local rectangular coordinate system (x, y, z) and the coordinate system (ν, τ) perpendicular and parallel to the hollow boundary $\mathbb{C}$ are also used as shown in Fig. 1.

From Maxwell's equations in the toroidal coordinate system, we can express E_r, E_θ, H_r, and H_θ by E_z and H_z in the hollow core region with a refractive index of n_0 as follows [7]:

$$E_r=-j\frac{1}{n_0^2k_0^2\left(1+\frac{r}{R}\cos\theta\right)^2-\beta^2}\left(n_0k_0\frac{\partial E_z}{\partial r}+\frac{\omega\mu_0}{r}\frac{\partial H_z}{\partial\theta}\right) \tag{1}$$

$$E_\theta=-j\frac{1}{n_0^2k_0^2\left(1+\frac{r}{R}\cos\theta\right)^2-\beta^2}\left(\frac{n_0k_0}{r}\frac{\partial E_z}{\partial\theta}-\omega\mu_0\frac{\partial H_z}{\partial r}\right) \tag{2}$$

$$H_r=-\frac{n_0k_0}{\omega\mu_0}E_\theta \tag{3}$$

$$H_\theta=\frac{n_0k_0}{\omega\mu_0}E_r \tag{4}$$

where the time and z dependences of the form $\exp j(\omega t-\beta z)$ are suppressed, and it is assumed that a characteristic length of the waveguide, say the core diameter, is sufficiently large, and $\beta\simeq n_0k_0$. It should be noted that β cannot be simply replaced by n_0k_0 when the term $\beta-n_0k_0$ appears, as shown in (10). The axial field components E_z and H_z can be determined from

$$\frac{1}{r}\frac{\partial}{\partial r}(rE_r)+\frac{1}{r}\frac{\partial E_\theta}{\partial\theta}=jn_0k_0E_z \tag{5}$$

$$\frac{1}{r}\frac{\partial}{\partial r}(rE_\theta)-\frac{1}{r}\frac{\partial E_r}{\partial\theta}=-j\omega\mu_0H_z \tag{6}$$

by substituting (1) and (2) into (5) and (6).

Expanding electric and magnetic fields $\mathbb{E}$ (E_r, E_θ, E_z), $\mathbb{H}$ (H_r, H_θ, H_z), and β as

$$\mathbb{E}=\mathbb{E}^{(0)}+\frac{1}{R}\mathbb{E}^{(1)}+\frac{1}{R^2}\mathbb{E}^{(2)}+\cdots \tag{7}$$

$$\mathbb{H}=\mathbb{H}^{(0)}+\frac{1}{R}\mathbb{H}^{(1)}+\frac{1}{R^2}\mathbb{H}^{(2)}+\cdots \tag{8}$$

$$\beta=\beta_0+\frac{1}{R}\delta\beta_1+\frac{1}{R^2}\delta\beta_2+\cdots \tag{9}$$

and noticing that the denominators of (1) and (2) can be approximated by including terms up to order R^{-2}

$$\left[n_0^2k_0^2\left(1+\frac{r}{R}\cos\theta\right)^2-\beta^2\right]^{-1}=\left(\frac{T}{u}\right)^2\left\{1-2\left(\frac{n_0k_0T}{u}\right)^2\cdot\left(r\cos\theta-\frac{\delta\beta_1}{n_0k_0}\right)\frac{1}{R}+2\left(\frac{n_0k_0T}{u}\right)^2\left[\frac{\delta\beta_2}{n_0k_0}+2\left(\frac{n_0k_0T}{u}\right)^2\cdot\left(r\cos\theta-\frac{\delta\beta_1}{n_0k_0}\right)^2\right]\frac{1}{R^2}\right\} \tag{10}$$

one can express $E_r^{(i)}$ and $E_\theta^{(i)}$ $(i=0,1,2)$ as follows:

$$E_r^{(i)}=2\left(\frac{n_0k_0T}{u}\right)^2\frac{\delta\beta_2}{n_0k_0}E_r^{(i-2)}-2\left(\frac{n_0k_0T}{u}\right)^2\cdot\left(r\cos\theta-\frac{\delta\beta_1}{n_0k_0}\right)E_r^{(i-1)}-j\left(\frac{T}{u}\right)^2\left[n_0k_0\frac{\partial E_z^{(i)}}{\partial r}+\frac{\omega\mu_0}{r}\frac{\partial H_z^{(i)}}{\partial\theta}\right] \tag{11}$$

$$E_\theta^{(i)} = 2\left(\frac{n_0 k_0 T}{u}\right)^2 \frac{\delta\beta_2}{n_0 k_0} E_\theta^{(i-2)} - 2\left(\frac{n_0 k_0 T}{u}\right)^2 \cdot \left(r\cos\theta - \frac{\delta\beta_1}{n_0 k_0}\right) E_\theta^{(i-1)} - j\left(\frac{T}{u}\right)^2 \left[\frac{n_0 k_0}{r}\frac{\partial E_z^{(i)}}{\partial\theta} - \omega\mu_0 \frac{\partial H_z^{(i)}}{\partial r}\right] \qquad (12)$$

where u is the transverse phase constant in the hollow region defined by

$$u^2 = \left(n_0^2 k_0^2 - \beta_0^2\right) T^2 \qquad (13)$$

and T is a characteristic length, say the core radius. Quantities with negative superscripts in (11) and (12) are understood to be zero. In (10), it is already taken into account that $|\delta\beta_1|^2$ is much smaller than $|2\beta_0\delta\beta_2|$ [14]. Equations (11) and (12) can be transformed to (see Appendix)

$$E_\nu^{(i)} = 2\left(\frac{n_0 k_0 T}{u}\right)^2 \frac{\delta\beta_2}{n_0 k_0} E_\nu^{(i-2)} - 2\left(\frac{n_0 k_0 T}{u}\right)^2 \left(r\cos\theta - \frac{\delta\beta_1}{n_0 k_0}\right) E_\nu^{(i-1)} - j\left(\frac{T}{u}\right)^2 \left[n_0 k_0 \frac{\partial E_z^{(i)}}{\partial\nu} + \omega\mu_0 \frac{\partial H_z^{(i)}}{\partial\tau}\right] \qquad (14)$$

$$E_\tau^{(i)} = 2\left(\frac{n_0 k_0 T}{u}\right)^2 \frac{\delta\beta_2}{n_0 k_0} E_\tau^{(i-2)} - 2\left(\frac{n_0 k_0 T}{u}\right)^2 \left(r\cos\theta - \frac{\delta\beta_1}{n_0 k_0}\right) E_\tau^{(i-1)} - j\left(\frac{T}{u}\right)^2 \left[n_0 k_0 \frac{\partial E_z^{(i)}}{\partial\tau} - \omega\mu_0 \frac{\partial H_z^{(i)}}{\partial\nu}\right]. \qquad (15)$$

Substitution of (11) and (12) into (5) and (6) leads to the differential equations for $E_z^{(i)}$ and $H_z^{(i)}$ $(i = 0,1,2)$ as follows:

$$\nabla^2 E_z^{(i)} + \left(\frac{u}{T}\right)^2 E_z^{(i)} = 2 n_0 k_0 \delta\beta_2 E_z^{(i-2)} - 2 n_0^2 k_0^2 \left(r\cos\theta - \frac{\delta\beta_1}{n_0 k_0}\right) E_z^{(i-1)} + j2 n_0 k_0 E_x^{(i-1)} \qquad (16)$$

$$\nabla^2 H_z^{(i)} + \left(\frac{u}{T}\right)^2 H_z^{(i)} = 2 n_0 k_0 \delta\beta_2 H_z^{(i-2)} - 2 n_0^2 k_0^2 \left(r\cos\theta - \frac{\delta\beta_1}{n_0 k_0}\right) H_z^{(i-1)} + j2 n_0 k_0 H_x^{(i-1)} \qquad (17)$$

where E_x and H_x are simply calculated by E_r, E_θ, and H_r, H_θ, respectively. Equations (16) and (17) should be integrated with the boundary conditions at $\mathbb{C}$ as

$$\frac{E_\tau^{(i)}}{H_z^{(i)}} = \frac{\omega\mu_0}{n_0 k_0} z_{\mathrm{TE}} \qquad (18)$$

$$\frac{H_\tau^{(i)}}{E_z^{(i)}} = -\frac{n_0 k_0}{\omega\mu_0} y_{\mathrm{TM}} \qquad (19)$$

where z_{TE} and y_{TM} are the normalized surface impedance and admittance, respectively [8].

We first mention the evaluation method of $\delta\beta_1$ by using only $\mathbb{E}^{(0)}$ and $\mathbb{H}^{(0)}$, i.e., field distributions in the straight waveguide.

Constructing

$$E_z^{(1)} \nabla^2 E_z^{(0)} - E_z^{(0)} \nabla^2 E_z^{(1)} \qquad (20)$$

using (16) and integrating in the hollow region, one obtains

$$\oint \left[E_z^{(1)} \frac{\partial E_z^{(0)}}{\partial\nu} - E_z^{(0)} \frac{\partial E_z^{(1)}}{\partial\nu}\right] dC = 2 n_0^2 k_0^2 \int \left(r\cos\theta - \frac{\delta\beta_1}{n_0 k_0}\right) E_z^{(0)^2}\, dS - j2 n_0 k_0 \int E_x^{(0)} E_z^{(0)}\, dS. \qquad (21)$$

Substituting $\partial E_z^{(0)}/\partial\nu$ and $\partial E_z^{(1)}/\partial\nu$ obtained from (14) into (21) and using the boundary condition (19), one obtains

$$\delta\beta_1 \left[\int E_z^{(0)^2}\, dS + j\frac{1}{\omega\epsilon_0 n_0^2} \oint E_z^{(0)} H_\tau^{(0)}\, dC\right] = n_0 k_0 \left\{\int r\cos\theta E_z^{(0)^2}\, dS - j\frac{1}{\omega\epsilon_0 n_0^2} \int E_z^{(0)} H_y^{(0)}\, dS + j\frac{1}{\omega\epsilon_0 n_0^2} \oint r\cos\theta E_z^{(0)} H_\tau^{(0)}\, dC\right\} + \frac{1}{2\omega\epsilon_0 n_0^2} \oint \left[E_z^{(1)} \frac{\partial H_z^{(0)}}{\partial\tau} - E_z^{(0)} \frac{\partial H_z^{(1)}}{\partial\tau}\right] dC. \qquad (22)$$

Similarly, integrating $H_z^{(0)} \nabla^2 H_z^{(1)} - H_z^{(1)} \nabla^2 H_z^{(0)}$ in the hollow region, one obtains

$$\delta\beta_1 \left[\int H_z^{(0)^2}\, dS - j\frac{1}{\omega\mu_0} \oint E_\tau^{(0)} H_z^{(0)}\, dC\right] = n_0 k_0 \left\{\int r\cos\theta H_z^{(0)^2}\, dS + j\frac{1}{\omega\mu_0} \int E_y^{(0)} H_z^{(0)}\, dS - j\frac{1}{\omega\mu_0} \oint r\cos\theta E_\tau^{(0)} H_z^{(0)}\, dC\right\} + \frac{1}{2\omega\mu_0} \oint \left[H_z^{(0)} \frac{\partial E_z^{(1)}}{\partial\tau} - H_z^{(1)} \frac{\partial E_z^{(0)}}{\partial\tau}\right] dC. \qquad (23)$$

By forming $\omega\epsilon_0 n_0^2 \times (22) + \omega\mu_0 \times (23)$, and using

$$\oint \left[E_z^{(1)} \frac{\partial H_z^{(0)}}{\partial\tau} - E_z^{(0)} \frac{\partial H_z^{(1)}}{\partial\tau}\right] dC + \oint \left[H_z^{(0)} \frac{\partial E_z^{(1)}}{\partial\tau} - H_z^{(1)} \frac{\partial E_z^{(0)}}{\partial\tau}\right] dC = \oint \frac{\partial}{\partial\tau} \left[E_z^{(1)} H_z^{(0)} - E_z^{(0)} H_z^{(1)}\right] dC = 0 \qquad (24)$$

we arrive at

$$\delta\beta_1\left\{\int\left[\omega\epsilon_0 n_0^2 E_z^{(0)^2}+\omega\mu_0 H_z^{(0)^2}\right]dS + j\oint\left[E_z^{(0)}H_\tau^{(0)}-E_\tau^{(0)}H_z^{(0)}\right]dC\right\}$$
$$= n_0k_0\left\{\int r\cos\theta\left[\omega\epsilon_0 n_0^2 E_z^{(0)^2}+\omega\mu_0 H_z^{(0)^2}\right]dS + j\int\left[E_y^{(0)}H_z^{(0)}-E_z^{(0)}H_y^{(0)}\right]dS + j\oint r\cos\theta\left[E_z^{(0)}H_\tau^{(0)}-E_\tau^{(0)}H_z^{(0)}\right]dC\right\}. \quad (25)$$

Equation (25) shows that $\delta\beta_1$ can be evaluated from only the zeroth-order fields $\mathbb{E}^{(0)}(E_\tau^{(0)}, E_y^{(0)}, E_z^{(0)})$ and $\mathbb{H}^{(0)}(H_\tau^{(0)}, H_y^{(0)}, H_z^{(0)})$, which makes the evaluation of $\delta\beta_1$ much simpler compared with the conventional method requiring the first-order perturbation terms of $\mathbb{E}^{(1)}$ and $\mathbb{H}^{(1)}$. It is clear that $\delta\beta_1=0$ when the waveguide is symmetric with respect to the plane $x=0$.

In the asymmetric three-layered slab waveguide, i.e., $\delta\beta_1\neq 0$, it was shown that the axial phase constant β should properly be described by using $\delta\beta_1$ as well as $\delta\beta_2$ [14]. Furthermore, since we intend to extend results for the bent circular waveguides to any waveguide, we proceed to the evaluation of $\delta\beta_2$.

Following a process similar to that used to obtain $\delta\beta_1$, i.e., integrating

$$E_z^{(0)}\nabla^2E_z^{(2)}-E_z^{(2)}\nabla^2E_z^{(0)} \quad (26)$$

and

$$H_z^{(0)}\nabla^2H_z^{(2)}-H_z^{(2)}\nabla^2H_z^{(0)} \quad (27)$$

in the hollow region, one obtains

$$\delta\beta_2\left[\int E_z^{(0)^2}dS + j\frac{1}{\omega\epsilon_0 n_0^2}\oint E_z^{(0)}H_\tau^{(0)}dC\right]$$
$$= n_0k_0\left\{\int\left[\left(r\cos\theta-\frac{\delta\beta_1}{n_0k_0}\right)E_z^{(0)}E_z^{(1)} - j\frac{1}{\omega\epsilon_0 n_0^2}H_y^{(1)}E_z^{(0)}\right]dS + j\frac{1}{\omega\epsilon_0 n_0^2}\oint\left(r\cos\theta-\frac{\delta\beta_1}{n_0k_0}\right)E_z^{(0)}H_\tau^{(0)}dC\right\}$$
$$+\frac{1}{2\omega\epsilon_0 n_0^2}\oint\left[E_z^{(2)}\frac{\partial H_z^{(0)}}{\partial\tau}-E_z^{(0)}\frac{\partial H_z^{(2)}}{\partial\tau}\right]dC \quad (28)$$

and

$$\delta\beta_2\left[\int H_z^{(0)^2}dS - j\frac{1}{\omega\mu_0}\oint H_z^{(0)}E_\tau^{(0)}dC\right] = n_0k_0\left\{\int\left[\left(r\cos\theta-\frac{\delta\beta_1}{n_0k_0}\right)H_z^{(0)}H_z^{(1)} + j\frac{1}{\omega\mu_0}E_y^{(1)}H_z^{(0)}\right]dS - j\frac{1}{\omega\mu_0}\oint\left(r\cos\theta-\frac{\delta\beta_1}{n_0k_0}\right)H_z^{(0)}E_\tau^{(0)}dC\right\} + \frac{1}{2\omega\mu_0}\oint\left[H_z^{(0)}\frac{\partial E_z^{(2)}}{\partial\tau}-H_z^{(2)}\frac{\partial E_z^{(0)}}{\partial\tau}\right]dC. \quad (29)$$

Therefore, by making $\omega\epsilon_0 n_0^2\times(28)+\omega\mu_0\times(29)$, one finally obtains the expression of $\delta\beta_2$ as follows:

$$\delta\beta_2\left\{\int\left[\omega\epsilon_0 n_0^2 E_z^{(0)^2}+\omega\mu_0 H_z^{(0)^2}\right]dS + j\oint\left[E_z^{(0)}H_\tau^{(0)}-H_z^{(0)}E_\tau^{(0)}\right]dC\right\}$$
$$= n_0k_0\left\{\int\left(r\cos\theta-\frac{\delta\beta_1}{n_0k_0}\right)\left[\omega\epsilon_0 n_0^2 E_z^{(0)}E_z^{(1)} + \omega\mu_0 H_z^{(0)}H_z^{(1)}\right]dS + j\int\left[E_y^{(1)}H_z^{(0)}-H_y^{(1)}E_z^{(0)}\right]dS + j\oint\left(r\cos\theta-\frac{\delta\beta_1}{n_0k_0}\right)\left[E_z^{(0)}H_\tau^{(1)}-H_z^{(0)}E_\tau^{(1)}\right]dC\right\}. \quad (30)$$

For circular waveguides, it is clear that (30) reduces to the result obtained previously [8].

The bending losses of the waveguides, i.e., the attenuation constants α of the modes in the curved waveguides, are simply evaluated by

$$\alpha = -I_m\left(\beta_0+\frac{\delta\beta_1}{R}+\frac{\delta\beta_2}{R^2}\right) \quad (31)$$

to the order of R^{-2}.

Finally, we mention the validity of (30) or (31). For the electric and magnetic fields, (7) and (8), obtained by the perturbation theory to describe the actual fields properly, it is necessary that the bending radius R is sufficiently large and the zeroth-order solutions $\mathbb{E}^{(0)}$ and $\mathbb{H}^{(0)}$ are much larger than the first-order solutions $\mathbb{E}^{(1)}$ and $\mathbb{H}^{(1)}$. For the circular waveguides [8], the above condition leads to

$$R \gg R_l \quad (32)$$

where R_l is defined by

$$|E_z^{(0)}|_{\max}=\frac{1}{R_l}|E_z^{(1)}|_{\max} \quad \text{or} \quad |H_z^{(0)}|_{\max}=\frac{1}{R_l}|H_z^{(1)}|_{\max} \quad (33)$$

and it was shown that the attenuation constant can be properly predicted by the present method even when R approaches R_l [15]. Therefore, we can expect that the conditions (32) and (33) are necessary for the present series approach to be valid.

III. Conclusion

A method for evaluating the complex propagation constant has been developed for oversized, bent, hollow waveguides with arbitrary cross sections. The method simplifies loss calculations relative to the conventional method, as was shown for the special case of circular waveguides [8].

Appendix

Let the angle between ν and r be ϕ, one can express E_ν and E_τ as follows:

$$\begin{pmatrix} E_\nu \\ E_\tau \end{pmatrix} = \begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix} \begin{pmatrix} E_r \\ E_\theta \end{pmatrix}. \tag{A1}$$

For an arbitrary scalar function F, we obtain

$$\begin{pmatrix} \dfrac{\partial F}{\partial \nu} \\ \dfrac{\partial F}{\partial \tau} \end{pmatrix} = \begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix} \begin{pmatrix} \dfrac{\partial F}{\partial r} \\ \dfrac{1}{r}\dfrac{\partial F}{\partial \theta} \end{pmatrix}. \tag{A2}$$

Therefore, by making

$$\text{Eq. (11)} \times \cos\phi - \text{Eq. (12)} \times \sin\phi \tag{A3}$$

$$\text{Eq. (11)} \times \sin\phi + \text{Eq. (12)} \times \cos\phi \tag{A4}$$

we finally obtain (14) and (15), respectively.

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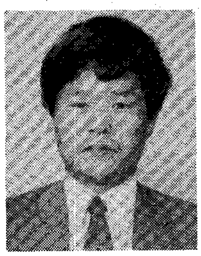

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